PH. D. Entrance examination syllabus-2020(MATHEMATICS)

SUBJECT SPECIFIC (MATHEMATICS)

1. Measure Theory

Lebesgue measure
Introduction, outer measure, measurable sets and Lebesgue measure, Anon measurable sets, measurable function, Littlewood's three principles.

Lebesgue integral
The Riemann integral, The lebesgue integral of a bounded function over a set of finite measure, The integral of a non negative function, The general Lebesgue integral, Lebesgue convergence theorem ,monotone convergence theorem, Fatau's lemma, generalisedlebesgue convergence theorem.

Differentiation and integration
Differentiation of monotone functions, functions of a bounded variation, Differentiation of an integral, absolute continuity, convex functions.

2. COMPLEX ANALYSIS

Complex Numbers:

Definition of Complex Numbers, Square roots, Geometric interpolation, Rational powers of a Complex Numbers, Topology of the complex plane, Sequence and series.

Analytic Functions:

Functions, limits and continuity, differentiability Power series as an analytic function, Exponential and Trigenometric functions, Complex logarithms, Inverse functions, Zeros of analytic functions.

Complex Integration:

Curves in the complex plane, Basic properties of complex Intergral, winding number or index of a curve, couchy-GaursatThorem, Homotopy version of Cauchy's theorem, Morea's theorem, Cauchy Integral Formula, Laurent series, The maximum modulus priniciple.

3. Functional Analysis

Metric Space:

Metric Space, Further Examples of Metric Spaces, Examples.Completeness proofs

Normed Spaces and Banach Spaces: Vector Space, Normed Space, Banach Space, Further Properties of Normed Spaces, FiniteDimensionalNormed Space and Subspaces, Compactness and Finite Dimension

Linear Operators: Bounded and Continuous Linear Operators, Linear Functionals, Linear Operators and Functionals on Finite Dimensional Spaces, Normed Spaces of Operators, Dual space.

4. Advanced Abstract Algebra

Group Theory:

Conjugate of an element, class equation, Cauchy theorem, First part of Sylow's theorem, Third part of Sylow's theorem, Application odSylow's theorem, Direct product of a group.

Ring Theory:

A particular Euclidean ring, Fermat's theorem, polynomial rings, primitive polynomials, Guass lemma, the Eisenstein criterion, polynomial rings over commutative rings, unique factorization domain.

Field Theory:

Extension fields, Finite extension field, Algebraic extension, Algebraic number, Roots of polynomials, splitting fields, Uniqueness of Splitting fields, construction with Straightedge and compass, More about roots, Simple extension, Fixed fields, Elementry symmetric functions, normal extension, Galois group, The fundamental theorem of Galois theory.

5. Advanced Linear Algebra

Linear operator

Functions, linear operators, null space and range, rank and nullity theorem, operator inverses, application to matrix theory, computation of null space and range of a matrix, matrix of an operator, change of basis and similar matrices.

Inner Product Spaces

Preliminaries: Definitions and examples, **Orthogonal sets**: Fourier coefficients and partial identity, gram-schmidt process, QR factorization, **Approximation and Orthogonal projection:** equivalence of the problems, Computations using orthogonal and nonorthogonal sets, normal equations, projection operators, $M^{\perp}$. **Orthogonal complements**: Decomposition of the vector space, applications to an approximations and matrix theory, **The Gram Matrix and Orthogonal Change of Basis:** matrix representation of an inner products, orthogonal change of bases, rank of gram matrix.

6. Ordinary Differential Equations

Basic concepts and Linear Equations of the first order: Introduction, Formation of Differential Equations (DE), Classification of DE, Initial and Boundary Value Problems, Definition of Solutions of DE, First Order Linear Equation, Exact Equations, Separable Equations.

Linear Differential Equations of Higher Order: Introduction, Higher Order Differential Equations, Linear Independence, Equations with Constant Coefficients, Equations with Variable Coefficients

Wronskian, Variation of Parameters, Some Standard Methods, Method of Laplace Transforms.

Solutions in Power Series:Introduction, Second Order Linear Equations with Ordinary Points, Legendre Equation and Legendre Polynomials, Second Order Equation with Regular Singular Point, Properties of Bessel Functions.

7. Numerical Analysis

Transcendental Equations:

Bisection method, Iteration method based on first and second degree equation, Rate of convergence, General iteration Methods, System of nonlinear equations, Methods for complex roots.

Polynomial Equations & System of Linear Algebraic Equations:

Polynomial equations, Choice of iterative method. Direct Methods, Error analysis for direct root methods, Iteration methods.

Eigen values problems:

Eigen values and Eigenvectors, Bound on Eigen values, Jacobi method for symmetric matrices, Givens method for arbitrary Matrices, Power method, Inverse power method.

Interpolation:

Lagrangian and Newton interpolations, Finite difference operators, Interpolating polynomials, Hermite interpolation, Piecewise and Spline interpolations.

8. Topology

Metric Spaces and Topological Spaces:

Sets and functions, Metric Spaces: Definitions and examples.

Topological Space, Relative Topology, Continuity and Convergence, Open and Closed sets, Closure of a set, Neighborhood of a point, Interior point, limit point, Derived set, Open Base and Open sub-base, First and second countable spaces, Separable spaces.

Compactness:

Cover, Sub-cover, open cover, Basic and sub-basic open cover, countable open cover, Continuity and Compactness, Finite intersection property, Heine-Borel property.

Product Spaces:

Definition and examples, Projection mappings and its continuity,Open and closed sub-base for product space.

Tychonoff's Theorem and Locally Compact Spaces:

Tychonoff's theorem, Generalized Heine-Borel theorem.

Compactness for Metric Spaces:

Sequentially Compact Metric Space, Bolzano-Weierstrass Property (BWP),Totally bounded Space,Ascoli's theorem.